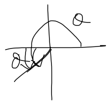
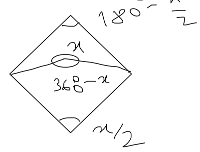
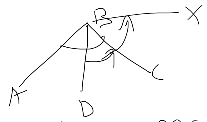


Lemma:- (Miquel Point of a Triangle)

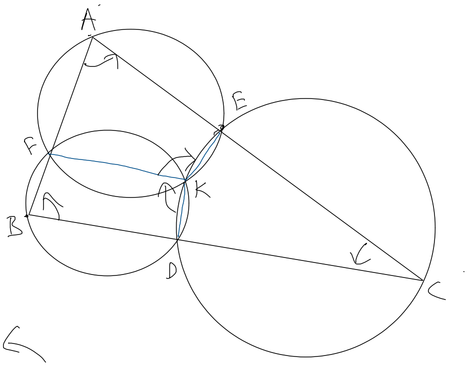
Points D, E, F lie on lines BC, CA, and AB of $\triangle ABC$, resp.
Then there exists a point lyin on the three circles (AEF), (BFD), (CDE).





$\measuredangle ABC + \measuredangle DBC = \measuredangle ABX$

$\measuredangle ABC + \measuredangle CBD = \measuredangle ABD$

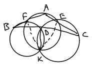


A, B, X, Y is cyclic $\Leftrightarrow \measuredangle AXB = \measuredangle AYB$

$\measuredangle FKE = -\measuredangle EKF$

$180^\circ + \measuredangle FKE = 180^\circ - \measuredangle EKF$

$\measuredangle FKE = 180^\circ - \measuredangle EKF$



$\angle FKE = \angle DKE + \angle FKD$

$\measuredangle DKE = \measuredangle DKF + \measuredangle FKE$

reason to use directed angles

Proof:- In cyclic quadrulateral AFKE, $\measuredangle FAE = \measuredangle FKE$
" " " FKBD, $\measuredangle DBF = \measuredangle DKF$

$$\measuredangle DKE = \measuredangle DBF + \measuredangle FAE = -\measuredangle ECD = \measuredangle DCE$$

$$\Rightarrow \measuredangle DKE = \measuredangle DCE \Rightarrow DKEC \text{ cyclic}$$

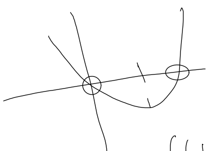
Q) If $\forall p \in \mathbb{R}$, one root of the equation $x^2 + 2px + q^2 - p^2 - 6 = 0$ is less than 1 and other root is greater than 1 then range of q is

Case 1:- $-\left(p + \sqrt{2p^2 - q^2 + 6}\right) < 1$
$-\left(p - \sqrt{2p^2 - q^2 + 6}\right) > 1$

Case 2:- $-\left(p + \sqrt{2p^2 - q^2 + 6}\right) > 1$
$-\left(p - \sqrt{2p^2 - q^2 + 6}\right) < 1$

used when p has a different range

Ans:- $\dfrac{-2p \pm \sqrt{4p^2 - 4q^2 + 4p^2 + 24}}{2}$



$f(1) < 0$

$1 + 2p + q^2 - p^2 - 6 < 0$

range of $p^2 - 2p + 5$
$(4, \infty)$

$(p-1)^2 + 4$

$1 + 2p + q^2 - p^2 - 6 < 0$

$q^2 < (p^2 - 2p + 5) \rightarrow \forall p \in \mathbb{R}$

$q^2 < (p-1)^2 + 4$

range of $p^2 - 2p + 5$

$(4, \infty)$

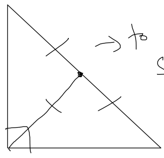
$(p-1)^2 + 4$

$q^2 < x \quad \forall \boxed{x \in (4, \infty)}$

$\Rightarrow q^2 < 4$

$\Rightarrow q \in (-2, 2)$

Q) In a ΔABC, let D be the midpoint of BC. If $\angle ADB = 45^\circ$ and $\angle ACD = 30^\circ$, determine $\angle BAD$.



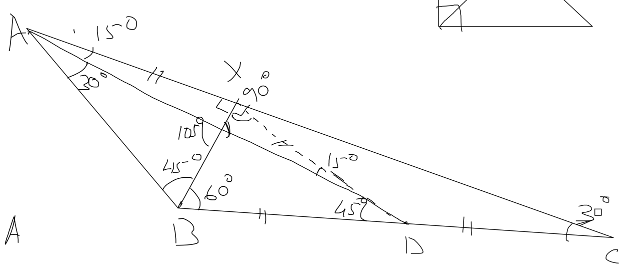
→ to be shown later

Ans:- $BD = DC$, $\angle ACD = 30^\circ$, $\angle ADB = 45^\circ$

$\angle CBX = 60^\circ$, $\angle BXC = 90^\circ$

$XD = BD = CD = XA = XB$

$\angle BXD = 60^\circ$, $\angle XDA = 15^\circ$



$XA = XB = XD \Rightarrow \angle XAB = \angle XBA$
$= 45^\circ$

$\Rightarrow \angle BAD = 30^\circ$